Robogals Science Challenge

Minor Challenge Set #1
STEM Field: Mathematics
Level: Senior
Challenge Name: A Travelling Problem
Project cost: 0-20 USD
Materials required:

- Pen and paper
- Printing (optional but convenient)

Duration:

- This challenge takes approximately an afternoon to a day to finish, however, the time guideline is an estimation only, and students and mentors can complete the tasks around their schedules.
- This problem has been designed so that you do not need to answer all the questions in one sitting. In fact, if you find a particular question too difficult, it is recommended that you take a break and re-attempt the problem with a fresher mind – after all, that's what mathematicians do when they solve hard problems – take breaks and think from different angles!

Introduction

In this Minor Challenge, we hope to introduce you to a historically notable problem in mathematics, and the field of graph theory. Graph theory is, as its name suggests, the study of graphs. In this context, a graph is made up of points (mathematical term: nodes) which are connected by lines (mathematical term: edges).

An application of graph theory is finding an optimal route to travel. In this case, the points (or nodes) are the locations we want to travel to, and the lines (or edges) are roads that we must travel on to reach the destinations. And so, graph theory can be used in GPS programming to find the best route to travel to the destination you put in.

Instruction

Note: You may find it easier to work with by printing out pages 2 to 6.

It is the 1700s and you are in the city of Königsberg in Prussia (now known as Kaliningrad, Russia). The city was set on both sides of the river Pregel, and included two islands that were connected to each other by seven bridges. The picture below shows the layout of the city.

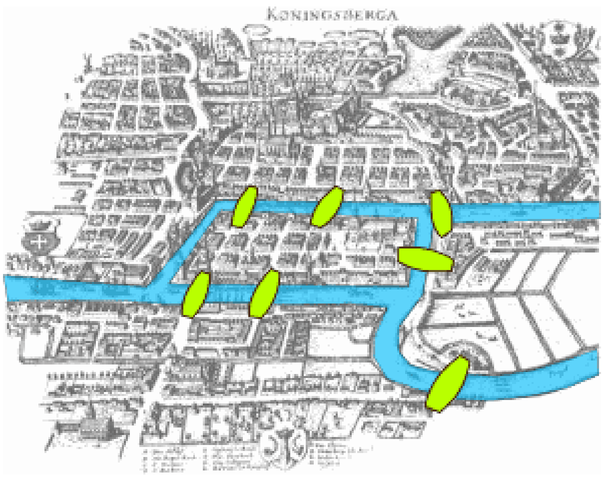


Figure 1: The map of Königsberg showing the layout of seven bridges in green and the river Pregel in blue.

To make it easier to solve this problem, we have transformed the map to a connected diagram.

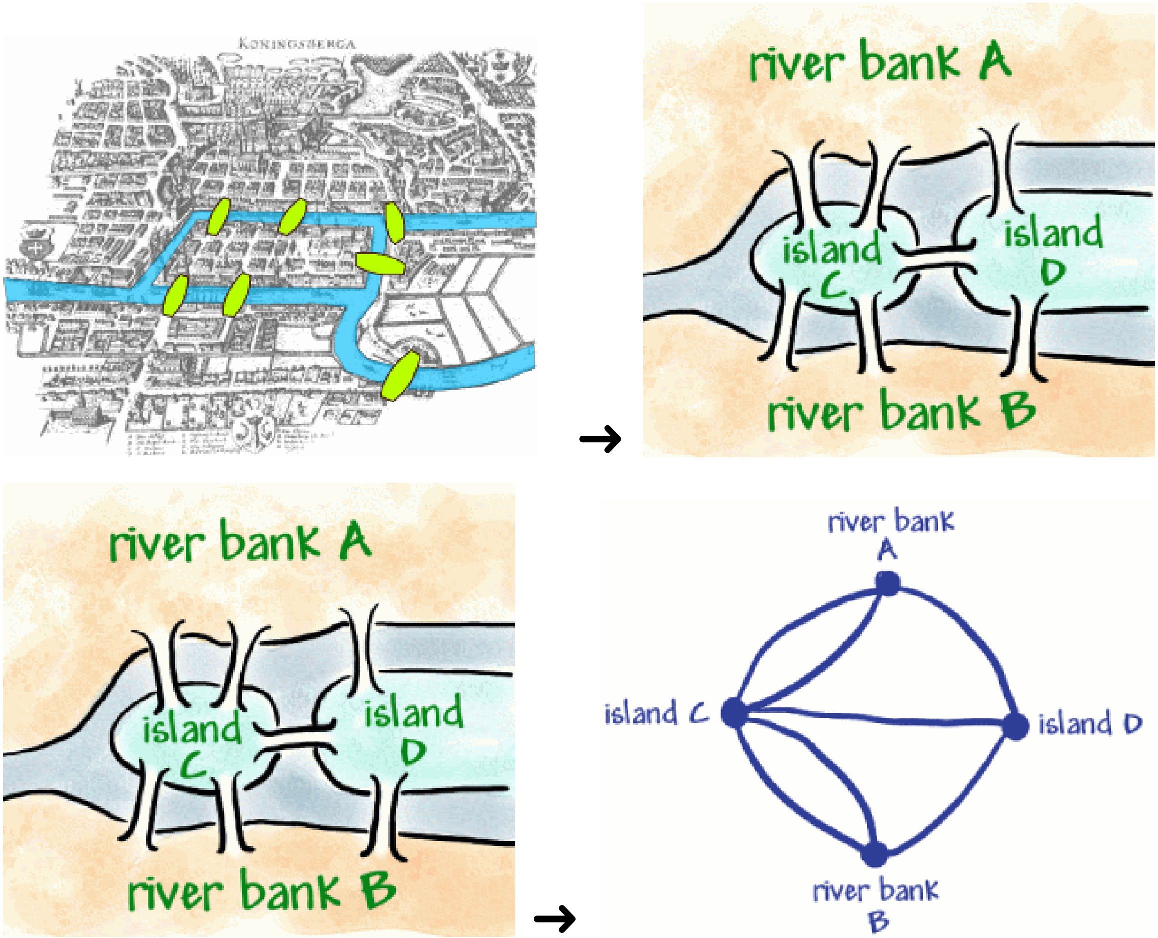


Figure 2: The transformation of the original map to a graph. In the final graph, the four locations are represented as dots with labels: river bank A, river bank B, island C, and island D. The bridges between the locations are represented as connecting lines.

You may have noticed that we distorted the original layout. The hint to this problem is that shapes of the landmasses and bridges do not matter. The important points are:

- Each location (two river banks and two islands) is represented by a dot
- Each bridge is represented by a line
- The number of bridges connected to each landmass stays the same

1. Can you find a way to travel around the town and only cross each bridge once? We recommend spending 15-30 minutes on this question, and note down your answer below. Don't worry if you haven't arrived at an answer yet!

   My solution:

2. If you remove any one bridge, can you find a solution to travel around the town and only cross each bridge once? Are there any restrictions to where your starting point is?

My solution:

You may have noticed that once you arrive at a dot, then unless it is the final dot where your walk ends, you need to leave the dot again via a line.

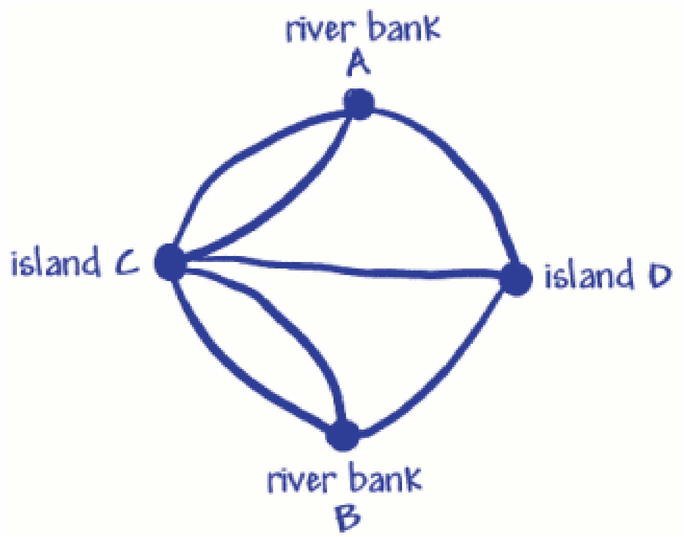


Figure 3: A graph showing the landmasses being represented as dots. These dots are labelled: river bank A, river bank B, island C, and island D. The bridges connecting the landmasses are represented by connecting lines.

3. Try adding a bridge between points A and D. Draw out your new network. Can you now find a route which crosses each edge only once? Are there any restrictions on where your starting point is?

My solution:

4. What did you notice about the graph? How many lines can you see are connected to each dot? Are the number of lines connected to each dot even numbers, or odd numbers?

My solution:

5. Can you add one (or more) edges or bridge to the original graph so that there is a walk which crosses every bridge exactly once, and where you finish at the same vertex where you started?

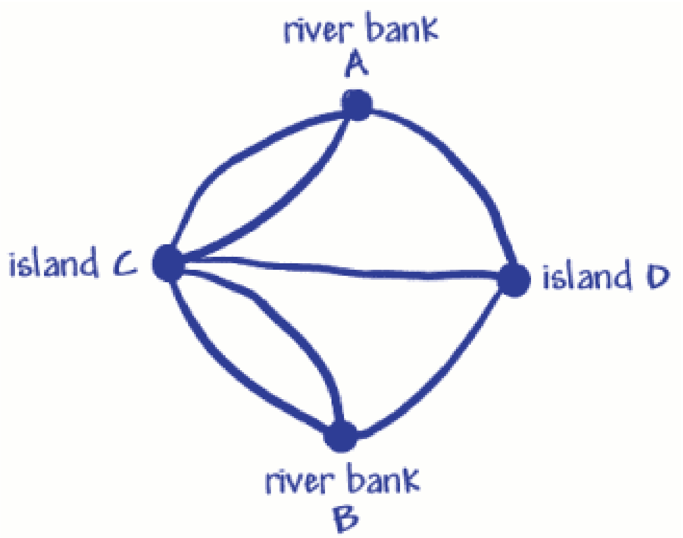


Figure 4: A graph showing the landmasses being represented as dots. These dots are labelled: river bank A, river bank B, island C, and island D. The bridges connecting the landmasses are represented by connecting lines.

My solution:

6. What did you learn from this activity? Here is an excellent video from TED discussing the problem and concepts of graph theory. https://ed.ted.com/lessons/how-the-konigsberg-bridge-problem-changed-mathematics-dan-van-der-vieren
   From your answers to questions 1-5 and watching the video, can you write 3-5 interesting points you learnt about the Bridge of Konigsberg and graph theory?

My solution:

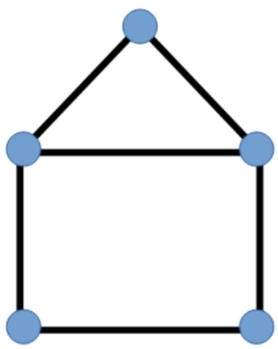
Extension

Task 1: Which network is traversable?

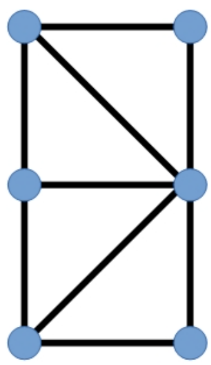
A **traversable** network is one you can draw without taking your pen off the paper, and without going over any edge twice.

For each network below, determine if the network is traversable.

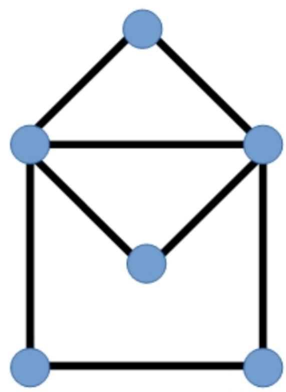
(1)



(2)



(3)



Task 2: The Four Colour Theorem

Graph theory is not just about determining traversable networks. One of the most famous theorems in mathematics – Four-Colour Theorem – relies on knowledge in graph theory. You may colour a graph's vertices, the edges, or the face of a graph.

Navigate to this website on your computer:
https://www.mathsisfun.com/activity/coloring.html

You will attempt to answer the question:
What is the fewest number of colours you need to colour a graph, such that two sections sharing a common edge cannot be coloured the same?

Reflection Questions

- Are there any improvements you would make to this challenge?
- What real world application can you apply this challenge to?

- Is this puzzle easier or harder than you expected?

- If you attempted the extension task, answer the following questions:
    - What do you notice about traversable networks where you started and finished in the same place?
    - From your observations, can you find a condition that guarantees a network is traversable?

Submission Guidelines

- Submit your answers for questions 1 to 6 and your answers to the reflection questions. If you attempted the extension task, please also include your solutions to the problems.

Note: Remember, if you want to upload pictures of your Minor Challenge that also include you, please check if it is OK with your mentor first.

- The submission form is on the Minor Challenges page: https://sciencechallenge.org.au/index.php/minor-challenges/ Fill out the details and make sure you upload your submission.

Learn More! Resources

- If you want to learn more on how the problem of the seven bridges relates to the modern life application of networks, this article has a detailed explanation.
  https://plus.maths.org/content/bridges-networks-0

- Here is a series of problems on network theory that helps you better understand the concept of networks.
  https://nrich.maths.org/453

Bibliography

- Nrich.maths.org. n.d. Can You Traverse It?. [online] Available at: <https://nrich.maths.org/11826> [Accessed 20 January 2022].
- plus.maths.org. 2013. Maths in a minute: The bridges of Königsberg. [online] Available at: <https://plus.maths.org/content/maths-minute-bridges-konigsberg> [Accessed 20 January 2022].
- Nrich.maths.org. n.d. The Bridges of Konigsberg. [online] Available at: <https://nrich.maths.org/2327?part=index> [Accessed 20 January 2022].